RAN-18030801010200001

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M.Sc. (Sem.-I) Examination November - 2025

PHYSICS : Classical Mechanics (PH-412)

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book

(2) Symbols Used have their usual meaning.

(3) Figures to the right indicate full marks.

Seat No.:

Student's Signature

Q.1. Attempt any two questions out of the (a), (b) and (c) below.

a. i. Prove Kepler's first law of planetary motion. **6**

ii. The maximum and minimum velocities of a satellite are V_{max} and V_{min} respectively. Prove that the eccentricity of the orbit of the satellite is $e = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$ **3**

b. i. Derive the equation of motion for a particle in a central force field. Also, derive first integrals and state their physical significance. **6**

ii. Sate the Bertrand's theorem. **2**

iii. In the potential $V = ar^{n+1}$, the force varies as the _____ power of r. **1**

c. i. Show how the two-body problem can be reduced to an equivalent one body problem using the concept of reduced mass. **5**

ii. Calculate the reduced mass (μ) of HCl. The atomic masses are approximately: **4**

- Hydrogen (H) = 1.00784 u
- Chlorine (CI) = 35.453 u

Where 1 u = 1.66053906660x 10^{-27} kg

Q.2. Attempt any two questions out of the (a), (b) and (c) below.

a. i. What are Euler's angles? Describe them **6**

ii. A rigid body consisting of N particles has at most _____ degrees of freedom before applying any constraints. **1**

The number of degrees of freedom of a rigid body is reduced from 9 to 6 because there are _____ constraint equations among the three reference points. **1**

The orientation of a rigid body can be specified by _____between the primed and unprimed coordinate axes. **1**

b. i. Discuss the eigenvalue of Inertia and Principle axis transformation. **5**

ii. Discuss the case of rotating earth and prove that precessional period of the earth is about 10 months. **4**

c. i. Explain torque-free motion of symmetrical top. **5**

ii. Calculate the inertia tensor for the system of four-point masses 1 gm, 2 gm, 3 gm and 4 gm, located at the point (1, 0, 0), (1, 1, 0), (1, 1, 1) and (l, 1, –l) cm. **4**

Q.3. Attempt any two question:

a. i. From the Taylor expansion of potential energy near equilibrium, derive Lagrangian for small oscillation and show that equation of motion for coupled oscillatory system reduces to $T_{ij}\ddot{n}_j + V_{ij}n_j = 0$. **6**

ii. What are normal coordinates? What is meant by normal modes of small oscillations? **3**

b. i. Generate Hamiltonian $H(q,p,t)$ by Legendre transformation from Lagrangian. $L(q, \dot{q}, t)$. Also right canonical equation of Hamilton. **5**

ii. Choosing the appropriate generalized coordinates write the Lagrangian and hence the Hamiltonian for the simple pendulum problem. Find the canonical equations of motion. **4**

c. i. What are the conditions under which the Hamiltonian of a system is equal to the total energy of the system? Should it always be constant? **5**

ii. Find the Routhian for the Lagrangian: L= $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$. **4**

Q.4. Attempt any two question :

a. i. Derive the canonical transformation relations for the generating function $F(q, Q, t)$. **5**

ii. If a Hamiltonian is given by $H = \frac{1}{2m}(p^2 + m^2\omega^2q^2)$, find new Hamiltonian $K(Q, P, t)$ via generating function $F_1 = \frac{m\omega q^2}{2}\cot Q$. **3**

b. i. Define Poisson brackets also evaluate the fundamental Poisson bracket. **5**

ii. State Jacobi identity and show that the Poisson brackets of two constants of motion is also a constant of motion. **3**

c. i. Define action-angle variables. What is the difference between vibration and Rotation type of periodic motions? **4**

ii. Solve the linear harmonic oscillator problem using Hamilton-Jacobi theory. **4**